

MUS 313 – Applied Tuning Systems
Steven M. Miller

Mathematics of Tuning Systems 2

Dividing Just Intervals

Just intervals cannot be divided equally (with a few special exceptions)

Superparticular ratios

Multiply numerator & denominator by x to create x new intervals

Interpolate new integers between them

Ex: divide $3/2$ into 3 parts: $3/2 * 2/2 = 6/4$

Interpolate 6:5:4; thus, a $3/2$ can be divided into a $6/5$ and $5/4$

Non-superparticular ratios:

Simply interpolate new integers

Ex: divide $5/3$ into 2 parts, interpolate 5:4:3; thus $5/4$ and $4/3$

Primary Intervals and Prime Limits

Each prime number has a *primary interval* associated with it

$P:2^n$; where p = prime number, and

2^n is the greatest power of 2 which is less than p

Ex: $2/1$, $3/2$, $5/4$, $7/4$, $11/8$, $13/8$, $19/16$

The *prime limit* is the highest prime factor in the fraction

$5/3$ = prime limit 5; $9/8$ = prime limit 3, etc.

All intervals in a scale in a prime limit are products of the primary intervals

Ex: 5 limit – $2/1$, $3/2$, $5/4$ (plus complements $4/3$ and $8/5$)

Exercises

1) Divide a $3/2$ into 4 intervals

2) Divide a $8/5$ into 3 intervals

3) Show that the following intervals are all products of 5-limit ratios

$1/1$ $16/15$ $9/8$ $6/5$ $5/4$ $4/3$ $64/45$ $3/2$ $8/5$ $5/3$ $16/9$ $15/8$ $2/1$

(hint, don't forget to octave-reduce and reduce to least-terms)

3) Show that the following intervals are all products of 7-limit ratios

$1/1$ $21/20$ $8/7$ $7/6$ $5/4$ $4/3$ $10/7$ $3/2$ $5/3$ $12/7$ $7/4$ $40/21$

(hint, don't forget to octave-reduce and reduce to least-terms)