

## MUS 313 – Applied Tuning Systems

### Steven M. Miller

## Mathematics of Tuning Systems 1

### Primes

Primes are integers evenly divisible only by themselves and 1  
1, 2, 3, 5, 7, 11, 13, 17, etc.

Any positive integer can be expressed as a product of primes

$$6=2*3$$

$$8=2*2*2$$

$$15=3*5$$

$$18=2*3*3$$

$$21=3*7$$

$$27=3*3*3$$

$$40=2*2*2*5$$

### Ratios

Reducing to least-terms

Cancel out common factors

Octave reduction

Numerator is larger than denominator

Make sure numerator is not more than twice the denominator

Divide numerator by 2's or multiply denominator by 2's

Prime limit

The 'prime limit' of a ratio is the highest prime factor in either the numerator or denominator

$$16/15 = (2*2*2*2)/(3*5); \text{ the prime limit is } 5$$

Intervals

Absolute

Relative to 1/1

Easily shows harmonic relationships

Adjacent

Relative to previous (lower) scale step

Easily shows melodic relationships

Scale pitches

Addition

Multiply ratios

Subtraction

Invert subtracted ratio and multiply

Complementation

Find the ratio that will equal 2/1 when added to current ratio

Subtract ratio from 2/1 to find complement; or

multiply denominator by 2 and invert

**Cents**

1 cent = 1/100 (1%) of an equal-tempered semitone

one octave = 1200 cents

$c = (1200/\log 2) * \log R$ ; where R = ratio in decimal form

**Absolute frequencies**

Convert to absolute ratios, then multiply ratios by starting frequency (1/1)

**Exercises**

1) Convert the following scale from absolute to adjacent interval ratios

1/1 16/15 9/8 6/5 5/4 4/3 64/45 3/2 8/5 5/3 16/9 15/8 2/1

(Terry Riley, *Harp of New Albion*)

2) Convert the following scale from adjacent to absolute interval ratios

16/15 25/24 21/20 15/14 16/15 25/24 27/25 16/15 25/24 21/20 15/14 16/15

(Lou Harrison, *Cinna*)

3) What is the prime limit for each of these two scales?

4) Determine the size, in ratios, of all fifths above each of the scale steps in #1

5) Once you have done exercise 2, convert all the absolute ratios to cents

6) Determine the cents deviation from 12-tet for #5

7) Determine the absolute frequencies for #1 based on 1/1 = 264 Hz.